

Gottfried sum rule in QCD NS analysis of DIS fixed target data

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Abstract

Deep inelastic scattering data on F_2 structure function obtained in the fixed-target experiments were analysed in the valence quark approximation with a next-to-next-to-leading-order accuracy. Parton distribution functions are parametrized by using information from the Gottfried sum rule. The strong coupling constant is found to be $\alpha_s(M_Z^2) = 0.1180 \pm 0.0020$ (total exp.error), which coincides very well with the average world value $\alpha_s^{\text{PDG}}(M_Z^2) = 0.1181 \pm 0.0013$ updated recently in a PDG report. The result for the second moment of the difference in u and d quark distributions $\langle x \rangle_{u-d} = 0.187 \pm 0.021$ is seen to be well compatible with the latest LATTICE result $\langle x \rangle_{u-d}^{\text{LATTICE}} = 0.208 \pm 0.024$.

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1 Introduction

The deep-inelastic lepton-hadron scattering (DIS) is a basic process for studying the properties of parton distribution functions (PDFs), which are universal in the considered scheme of calculations and can be used in the subsequent analyses of various processes.

The valence quark approximation¹ is a simplest one because of the absence of gluons. Due to the QCD factorization and evolution, there is significant correlation between the values of the strong coupling constant $\alpha_s(M_Z^2)$ and gluon density. That is why NS approximation is very well suited for evaluating the strong coupling constant, though it is of course cannot be considered complete.

The modern level of approximation is the next-to-next-to-leading order (NNLO) one. First NS analyses at the NNLO level [1] showed a small decrease in the average values of $\alpha_s(M_Z^2)$ with respect to the corresponding next-to-leading order (NLO) values $\alpha_s^{\text{NLO}}(M_Z^2)$. Similar behavior has been found also in other analyses (see, for example, [2] for a recent review).

We note that the NNLO analyses in [1] contained only BCDMS data, which as a rule led to somewhat low values of the strong coupling constant. Sometimes it is called a BCDMS effect. As it was recently shown in [3, 4] the BCDMS data [5, 6, 7] can be responsible for the large differences in both the cross-section values and extracted parameters observed in the analyses done by using Alekhin-Blumlein-Moch (ABM) [9] and Jimenez-Delgado-Reya (JR) PDF sets [10] on one hand and CTEQ [11], NN21 [12] and MSTW [13] ones on

¹Often it is called a nonsinglet (NS) approximation and so will we do.

the other.² Indeed, the results in [9, 10] were obtained by fitting mostly DIS data, while other groups [11, 12, 13] included in their fits some other experimental data (see [14] and references therein).

In [19] it was shown that those precise BCDMS data were collected with large systematic errors within certain ranges, which can presumably be responsible for an effective decrease in the value of $\alpha_s(M_Z^2)$ (see [19, 20, 21, 3, 4] and [22]).

The purpose of the present paper is to constrain PDFs by using the Gottfried sum rule [24] in the analyses and to evaluate the strong coupling constant and the second Mellin moment $\langle x \rangle_{u-d}$ in that case. The latter is a point of interest for the lattice QCD community (among other quantities of course); therefore, we compare it with their numbers.

2 Approach

As we have already mentioned in the Introduction, one of the most accurate processes to extract $\alpha_s(M_Z^2)$ is the valence part of DIS structure function (SF) F_2 , which is free of any correlations with the gluon density, hence the consideration limited to the valence part only.³

DIS structure function (SF) $F_2(x, Q^2)$ is dealt with by analyzing SLAC, NMC and BCDMS experimental data [5, 6, 7, 25, 26, 27] at NNLO of massless perturbative QCD. As in our previous papers the function $F_2(x, Q^2)$ is represented as a sum of the leading twist $F_2^{pQCD}(x, Q^2)$ and twist four terms

$$F_2(x, Q^2) = F_2^{pQCD}(x, Q^2) \left(1 + \frac{\tilde{h}_4(x)}{Q^2} \right), \quad (1)$$

where $F_2^{pQCD}(x, Q^2)$ denotes the twist-2 part with target mass corrections included. The second term $\sim \tilde{h}_4(x)$ denotes nonzero twist-4 corrections. For more details concerning an approach to analysing the experimental data we adopt refer to [20, 22].

2.1 Parton densities

The moments $\mathbf{f}_i(n, Q^2)$, ($i = \text{ns, q, g}$) at some Q_0^2 is a theoretical input to the analysis. In the fits of data with the cut $x \geq 0.25$ imposed, only the nonsinglet parton density is in the game and the following parametrization at the normalization point is used (see, for example, [21]):

$$\begin{aligned} \mathbf{f}(n, Q^2) &= \int_0^1 dx x^{n-2} \tilde{\mathbf{f}}(x, Q^2), \\ \tilde{\mathbf{f}}_{D,C}(x, Q^2) &= A_{D,C}(Q^2) [1-x]^{b_{D,C}(Q^2)} [1+d_{D,C}(Q^2)x], \\ \tilde{\mathbf{f}}_H(x, Q^2) &= \tilde{\mathbf{f}}_D(x, Q^2) + \frac{I_2}{N_H} x^{\lambda_H(Q^2)} [1-x]^{b_H(Q^2)} [1+d_H(Q^2)x], \end{aligned} \quad (2)$$

where $A(Q^2)$, $\lambda(Q^2)$, $b(Q^2)$ and $d(Q^2)$ are some coefficient functions. Here, the normalization

$$\begin{aligned} N_H &= \int_0^1 \frac{dx}{x} x^{\lambda_H(Q^2)} [1-x]^{b_H(Q^2)} [1+d_H(Q^2)x] \\ &= \frac{\Gamma(\lambda_H(Q^2)) \Gamma(1+b_H(Q^2))}{\Gamma(1+\lambda_H+b_H(Q^2))} \left[1 + d_H(Q^2) \frac{\lambda_H(Q^2)}{1+\lambda_H+b_H(Q^2)} \right] \end{aligned} \quad (3)$$

²The low values of $\alpha_s(M_Z^2)$ in [9, 10] can partially be explained [8] by the usage of the fixed flavor number scheme in deriving the ABM sets.

³As is usually done in such an approximation we restrict analysis to the large x region, nevertheless the data on the total structure function $F_2(x, Q^2)$ will be considered.

and the factor I_2 is related with the Gottfried sum rule I_G [24] (see also the review [44]) as follows

$$I_G(Q^2) = C_{\text{NS}}(a_s(Q^2))I_2(Q^2) = \left(1 + B_G a_s^2(Q^2)\right) I_2(Q^2), \quad (4)$$

where [28, 29, 30]

$$B_G \equiv B_{\text{NS}}^{(2)}(n=1) \approx -0.615732. \quad (5)$$

Note that the result (5) cannot be directly obtained in the standard calculations of F_2 Mellin moments through the optical theorem and the results for the forward cross-sections (see, for example, review [31] and discussions therein). The standard approach is to calculate only even Mellin moments while the result in (5) comes from the analytic continuation [32, 30] of the results for even Mellin moments or through an integration of the corresponding splitting functions in x -space (see [28, 29]), obtained in their turn from the even Mellin moments and some symmetric properties [28, 29]. Both approaches lead to the same results.

The Gottfried sum rule I_G has the form

$$I_G(Q^2) = 1 - 2 \int_0^1 \frac{dx}{x} \left(\mathbf{f}_{\bar{d}}(x, Q^2) - \mathbf{f}_{\bar{u}}(x, Q^2) \right), \quad (6)$$

Experimentally, at $Q_c^2 = 4 \text{ GeV}^2$ (see [43]),

$$I_G(Q_c^2) = 0.705 \pm 0.078, \quad (7)$$

i.e. it contains quite strong contribution from the second term, i.e. from the nonsymmetric sea.

2.2 Fitting procedure

As in all our previous papers on the subject we use the Jacobi polynomial expansion method (see, e.g. [21]). With the QCD expressions for the Mellin moments $M_n^k(Q^2)$ (see, for example, [22]) the SF $F_2^k(x, Q^2)$ is reconstructed by using the Jacobi polynomial expansion method:

$$F_2^k(x, Q^2) = x^a (1-x)^b \sum_{n=0}^{N_{\text{max}}} \Theta_n^{a,b}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M_{j+2}^k(Q^2),$$

where $\Theta_n^{a,b}$ are the Jacobi polynomials and a, b are the parameters to fit. A condition imposed on the latter is the requirement of the minimal error in reconstructing the structure functions. MINUIT package [42] is as usual used to minimize two variables; namely, the function F_2 itself and its logarithmic “slope” $d \ln F_2(x, Q^2) / d \ln \ln(Q^2/\Lambda^2)$. The twist expansion is thought to be applicable above approximately $Q^2 \sim 1 \text{ GeV}^2$ hence the global cut $Q^2 \geq 1 \text{ GeV}^2$ imposed throughout.

We use free normalization of the data for different experiments. For a reference set, the most stable deuterium BCDMS data at the value of the beam initial energy $E_0 = 200 \text{ GeV}$ is used. When other datasets are taken as a reference one, variation in the results is found to be negligible. In the case of the fixed normalization for each and all datasets the fits tend to yield a little bit worse χ^2 .

3 Q^2 -dependence of SF moments

Q^2 -dependence of the twist-2 part of the SF moments

$$M_n(Q^2) = \int_0^1 x^{n-2} F_2(x, Q^2) dx \quad (8)$$

has the following form

$$M_n^{NS}(Q^2) = R_{NS} \times \tilde{M}_n^{NS}(Q^2) \quad (9)$$

where R_{NS} is a normalization constant and the (perturbatively calculated) factor \tilde{M} contains the product of the coefficient function $C_{NS}(n, a_s(Q^2))$ and “renormgroup exponent” $h^{NS}(n, Q^2)$

$$\tilde{M}_n^{NS}(Q^2) = C_{NS}(n, a_s(Q^2)) h^{NS}(n, Q^2) \quad (10)$$

with

$$C_{NS}(n, a_s(Q^2)) = 1 + a_s(Q^2) B_{NS}^{(1)}(n) + a_s^2(Q^2) B_{NS}^{(2)}(n) + \mathcal{O}(a_s^3(Q^2)), \quad (11)$$

and

$$h^{NS}(n, Q^2) = a_s(Q^2)^{\frac{\gamma_{NS}^{(0)}(n)}{2\beta_0}} \left[1 + a_s(Q^2) Z_{NS}^{(1)}(n) + a_s^2(Q^2) Z_{NS}^{(2)}(n) + \mathcal{O}(a_s^3(Q^2)) \right], \quad (12)$$

where

$$\begin{aligned} Z_{NS}^{(1)}(n) &= \frac{1}{2\beta_0} \left[\gamma_{NS}^{(1)}(n) - \gamma_{NS}^{(0)}(n) b_1 \right], \quad b_i = \frac{\beta_i}{\beta_0}, \\ Z_{NS}^{(2)}(n) &= \frac{1}{4\beta_0} \left[\gamma_{NS}^{(2)}(n) - \gamma_{NS}^{(1)}(n) b_1 + \gamma_{NS}^{(0)}(n) (b_1^2 - b_2) \right] + \frac{1}{2} Z_{NS}^2(n). \end{aligned} \quad (13)$$

Here $\gamma_{NS}^{(k)}(n)$ are the factors in front of a_s in the expansion with respect to the latter of the anomalous dimensions $\gamma_{NS}(n, a_s)$.

The evolution of moments $M_n(f, Q^2)$ within an interval with the same value of f active quarks has a simple form [21]

$$\frac{M_n(f, Q_1^2)}{M_n(f, Q_2^2)} = \frac{\tilde{M}_n(f, Q_1^2)}{\tilde{M}_n(f, Q_2^2)} \quad (14)$$

4 Thresholds

Following [21], at the f -quark threshold $Q^2 = Q_f^2$ there is a smooth behavior of $F_2(x, Q^2)$ (1) in the nonsinglet approximation as well as its moments (here the factor f marks the region with f active quarks)

$$M_n(f, Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2), \quad (15)$$

with

$$M_n(f, Q_f^2) = M_n(f-1, Q_f^2) \quad (16)$$

Thus, if Q^2 and Q_0^2 values belong to the intervals with, e.g., $f = 4$ and $f = 5$, respectively, then the evolution of I_G from Q_0^2 to Q^2 should contain the following factor

$$\frac{M_n(f=4, Q^2)}{M_n(f=4, Q_{f=5}^2)} \frac{M_n(f=5, Q_{f=5}^2)}{M_n(f=5, Q_0^2)} \quad (17)$$

which, according to (16), simplifies to

$$\frac{M_n(f=4, Q^2)}{M_n(f=5, Q_0^2)} = \frac{\tilde{M}_n(f=4, Q^2)}{\tilde{M}_n(f=5, Q_0^2)} \quad (18)$$

and the coupling constant acquires additional terms (see [45] and references therein)

$$\frac{a_s(f-1, Q_f^2)}{a_s^2(f, Q_f^2)} = 1 - \frac{2}{3} l_f a_s(f, Q_f^2) + \frac{4}{3} \left(l_f^2 - \frac{57}{2} l_f + \frac{11}{2} \right) a_s^2(f, Q_f^2) \quad (19)$$

$$\frac{a_s(f, Q_f^2)}{a_s^2(f-1, Q_f^2)} = 1 + \frac{2}{3} l_f a_s(f-1, Q_f^2) + \frac{4}{3} \left(l_f^2 + \frac{57}{2} l_f - \frac{11}{2} \right) a_s^2(f-1, Q_f^2) \quad (20)$$

with

$$l_f = \ln(Q_f^2/m_f^2) \quad (21)$$

where m_f is the mass of f -flavor quark.

Quite the same way, if Q^2 and Q_0^2 values belong to the intervals with $f = 3$ and $f = 5$, respectively, then the evolution of M_n from Q_0^2 to Q^2 should contain the following factor

$$\frac{M_n(f=3, Q^2)}{M_n(f=3, Q_{f=4}^2)} \frac{M_n(f=4, Q_{f=4}^2)}{M_n(f=4, Q_{f=5}^2)} \frac{M_n(f=5, Q_{f=5}^2)}{M_n(f=5, Q_0^2)} \quad (22)$$

which, according to (16), simplifies to

$$\frac{M_n(f=3, Q^2)}{M_n(f=5, Q_0^2)} = \frac{\tilde{M}_n(f=3, Q^2)}{\tilde{M}_n(f=5, Q_0^2)}. \quad (23)$$

In other words, the values of the moments at thresholds do not contribute. They always cancel each other out according to (16)⁴.

In general, if Q^2 (Q_0^2) value belongs to the interval with f (f_0) active quarks, the evolution is found to be very simple

$$\frac{M_n(f, Q^2)}{M_n(f_0, Q_0^2)} = \frac{\tilde{M}_n(f, Q^2)}{\tilde{M}_n(f_0, Q_0^2)}. \quad (24)$$

Turning back to the Gottfried sum rule we have that $I_G(f, Q^2) \equiv M_{n=1}(f, Q^2)$ is smooth at $Q^2 = Q_f^2$; therefore,

$$I_G(f, Q_f^2) = I_G(f-1, Q_f^2) \quad (25)$$

and the evolution has the form (if Q^2 (Q_0^2) value belongs to the interval with f (f_0) active quarks):

$$\frac{I_G(f, Q^2)}{I_G(f_0, Q_0^2)} = \frac{\tilde{M}_{n=1}(f, Q^2)}{\tilde{M}_{n=1}(f_0, Q_0^2)}, \quad (26)$$

Note that the values of \tilde{M}_n simplify at $n = 1$, since $B_{\text{NS}}^{(1)}(n = 1) = 0$ and $\gamma_{\text{NS}}^{(0)}(n = 1) = 0$.

Indeed,

$$C_{\text{NS}}(n = 1, a_s(Q^2)) = 1 + a_s^2(Q^2)B_G + \mathcal{O}(a_s^3(Q^2)). \quad (27)$$

and

$$h^{\text{NS}}(n, Q^2) = 1 + a_s(Q^2)d_1 + a_s^2(Q^2)[d_2 + (d_1 - b_1)d_1], \quad b_1 = \frac{\beta_1}{\beta_0}, \quad d_i = \frac{\gamma_{\text{NS}}^{(i)}(n = 1)}{2\beta_0}, \quad (28)$$

with [38, 30]

$$a_s(Q^2) = \frac{\alpha_s(Q^2)}{4\pi}, \quad \gamma_{\text{NS}}^{(1)}(n = 1) = \frac{8}{9}[13 + 8\zeta_3 - 12\zeta_2] \approx 7.28158, \\ \gamma_{\text{NS}}^{(2)}(n = 1) \approx 161.713785 - 2.429260f. \quad (29)$$

(see also Eqs. (4) and (5) given above).

Note that $I_2(Q^2)$ can be obtained at any Q^2 by inverting Eq. (4)

$$I_2(Q^2) = \frac{I_G(Q^2)}{1 + B_G a_s^2(Q^2)}, \quad (30)$$

where B_G is given in Eq. (5).

⁴This statement is correct exactly; however, in certain order of PT there is some difference between them (see, e.g., [21] for more detail)

5 Results

As the valence quark analysis does not deal with gluons the cut imposed on the Bjorken variable ($x \geq 0.25$) effectively excludes the region where gluon density is believed to be non-negligible. Concerning a twist expansion it is applicable only above $Q^2 \sim 1 \text{ GeV}^2$ hence the cut $Q^2 \geq 1 \text{ GeV}^2$ imposed on data throughout.

A starting point of the evolution is $Q_0^2 = 90 \text{ GeV}^2$. This Q_0^2 value is close to the average value of Q^2 spanning the respective data. The heavy quark thresholds are taken at $Q_f^2 = m_f^2$.

5.1 PDFs, strong coupling constant and high twist correction

As in [20, 21] the data with largest systematic errors are cut out by imposing certain limits on the kinematic variable Y . The cuts imposed are $x \geq 0.25$ and $N_{Y_{cut}} = 5$ (see Table 1 in [20, 21]). Then, a complete set of data consists of 756 points.

The following values for PDF parametrization (2) are obtained in the fits

in the case of the fixed $\lambda_H(Q^2) = 0.5$

$$\begin{aligned} A(D_2) &= 2.362 \pm 0.068, & A(C) &= 3.301 \pm 0.065, \\ b(H_2) &= 4.256 \pm 0.059, & b(D_2) &= 4.228 \pm 0.022, & b(C) &= 4.224 \pm 0.041, \\ d(H_2) &= 12.16 \pm 2.35, & d(D_2) &= 3.956 \pm 0.258, & d(C) &= 1.990 \pm 0.252. \end{aligned}$$

in the case of the free $\lambda_H(Q^2)$ values

$$\begin{aligned} \lambda_H &= 0.742 \pm 0.043, & A(D_2) &= 2.362 \pm 0.070, & A(C) &= 3.294 \pm 0.054, \\ b(H_2) &= 4.314 \pm 0.065, & b(D_2) &= 4.228 \pm 0.023, & b(C) &= 4.226 \pm 0.041, \\ d(H_2) &= 3.979 \pm 0.941, & d(D_2) &= 3.957 \pm 0.266, & d(C) &= 2.001 \pm 0.248. \end{aligned}$$

They are seen to be very similar to those presented in [21].

Table 1. *Parameter values of the twist-four term $\hat{h}_4(x)$*

x	$\lambda_H = 0.5$	free λ_H
0.275	-0.167 \pm 0.011	-0.167 \pm 0.013
0.35	-0.205 \pm 0.007	-0.205 \pm 0.007
0.45	-0.161 \pm 0.020	-0.162 \pm 0.020
0.55	-0.137 \pm 0.040	-0.138 \pm 0.040
0.65	-0.129 \pm 0.072	-0.132 \pm 0.074
0.75	-0.148 \pm 0.117	-0.148 \pm 0.121

Twist-4 parameter values are presented in Table 1. It is seen that they are almost the same for both cases and, moreover, are observed to be very similar to those given in [21].

Finally, using the nonsinglet evolution analyses of SLAC, NMC and BCDMS experimental data for SF F_2 for the fixed $\lambda_H = 0.5$ we obtain (with $\chi^2/DOF = 1.03$)

$$\alpha_s(M_Z^2) = 0.11795 + \left\{ \begin{array}{l} \pm 0.0004 \text{ (stat)} \pm 0.0018 \text{ (syst)} \pm 0.0006 \text{ (norm)} \\ \pm 0.0019 \text{ (total exp.error)} \end{array} \right. . \quad (31)$$

With free $\lambda_H(Q^2)$, the following result is obtained (with $\chi^2/DOF = 0.88$):

$$\alpha_s(M_Z^2) = 0.11798 + \left\{ \begin{array}{l} \pm 0.0003 \text{ (stat)} \pm 0.0019 \text{ (syst)} \pm 0.0005 \text{ (norm)} \\ \pm 0.0020 \text{ (total exp.error)} \end{array} \right. . \quad (32)$$

It is seen that the values of the strong coupling constant $\alpha_s(M_Z^2)$ are almost the same in the cases of fixed and free values of λ_H .

5.2 Second moment

The second moment $n = 2$ of the difference in valence parts of u and d quark distributions is also investigated in the lattice models. Following [15, 40], we will estimate this second moment illustrating some features of the nucleon structure.

This difference in valence u and d quark distributions can be extracted at large x values directly from that of the nonsinglet parton densities in proton and deuteron

$$\tilde{\mathbf{f}}_u^v(x, Q^2) - \tilde{\mathbf{f}}_d^v(x, Q^2) \approx \tilde{\mathbf{f}}_p(x, Q^2) - \tilde{\mathbf{f}}_d(x, Q^2), \quad (33)$$

since a contribution of the sea quarks and antiquarks are negligible here (see, for example, recent paper [47]).

Thus, using results given in (31) and (32) the respective difference for the second moments $\mathbf{f}_u^v(2, Q^2) - \mathbf{f}_d^v(2, Q^2)$ is shown in Table 6.

Table 6. The difference $\mathbf{f}_u^v(2, Q^2) - \mathbf{f}_d^v(2, Q^2)$.

Q^2 , GeV ²	90	4	2	1
$\lambda_H = 0.5$	0.110±0.012	0.139±0.016	0.150±0.017	0.179±0.020
free λ_H	0.115±0.013	0.145±0.016	0.157±0.017	0.187 ±0.021

Following [3], we would like to note that the lattice results are strongly nonperturbative and, therefore, it is better to compare them with our result obtained at $Q^2 = 1$ GeV², which corresponds in the present analysis to the boundary between perturbative and nonperturbative QCD. A comparison of our result (at $Q^2 = 1$ GeV²) with those derived in the lattice QCD approach is shown in Fig. 1. The lattice points are taken from the recent paper [23].

It is seen that the result of the analysis with free λ_H almost coincides with that presented in [3]. However, results presented here demonstrate considerably smaller uncertainties, which is a direct consequence of the new form (2) and (3) adopted in the PDF parametrizations.

6 Conclusions

In this work the Jacobi polynomial expansion method developed in [36] was used to analyze Q^2 -evolution of DIS structure function F_2 by fitting reliable fixed-target experimental data that satisfy the cut $x \geq 0.25$. Based on the results of fitting, the strong coupling constant is evaluated at the normalization point.

The present results are compatible with those obtained in our previous papers [21, 3]. Moreover, they almost coincide for both cases ($\lambda_H = 0.5$ fixed and free λ_H one). The only difference is in the uncertainties. For $\lambda_H = 0.5$ we have

$$\alpha_s(M_Z^2) = 0.1180 \pm 0.0019 \text{ (total exp.error)}, \quad (34)$$

With freely varied values of λ_H we obtain

$$\alpha_s(M_Z^2) = 0.1180 \pm 0.0020 \text{ (total exp.error)}. \quad (35)$$

Our result almost coincides with the central world average value

$$\alpha_s(M_Z^2)|_{\text{world average}} = 0.1181 \pm 0.0013, \quad (36)$$

presented in [48].

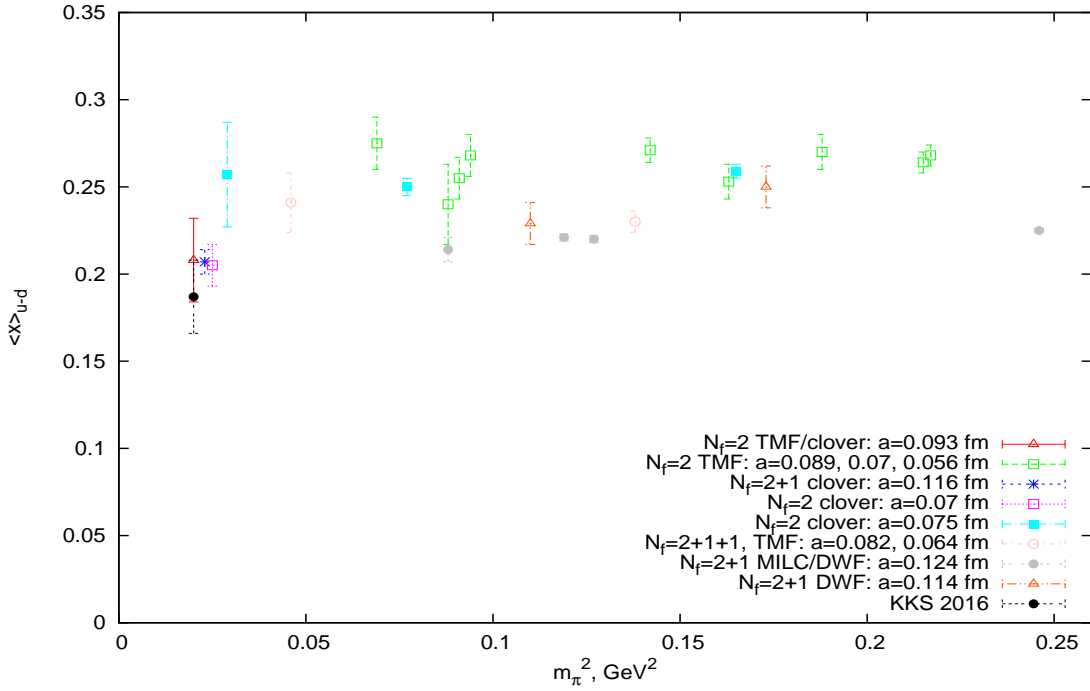


Figure 1: Lattice computations results for the second moment (or isovector nucleon momentum fraction) vs a pion mass m_π as borrowed from [23]; our KKS 2016 point for $\langle x \rangle_{u-d} \equiv \mathbf{f}_u^v(2, Q^2) - \mathbf{f}_d^v(2, Q^2)$ is obtained at $Q^2 = 1 \text{ GeV}^2$.

Complete agreement with a recent result obtained in lattice QCD [23] is also observed, which is exhibited by the results for the second moment $\langle x \rangle_{u-d}$ (see Fig. 1).

As the next steps of our investigations, we plan to perform N^3LO fits at large x values. In order to do that we use the contributions coming from three loops in the coefficient functions [49], as well as four-loop corrections to the first several moments of anomalous dimensions (see [50, 51, 52] and discussions therein). The knowledge of first several moments of anomalous dimensions is enough to do analyses of such a type. In a sense it is like the first NNLO analysis performed in [1]. We note that several N^3LO fits had already been done in [39, 40] with suggestion about a negligible contributions coming from the four-loop anomalous dimensions.

We plan to apply some resummations, like a Grunberg effective charge method [53] (as it was done in [37] at the NLO approximation) and the “frozen” and analytic modifications of the strong coupling constant (see [41] and references therein). Note that the resummations and the infrared safe modifications of the strong coupling constant often lead to the agreement between experimental data and theoretical predictions significantly improved (see [37, 54] and [41, 55], respectively, and references and discussions therein). It is hoped that similar properties will be observed in the forthcoming investigations.

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References

- [1] G. Parente, A. V. Kotikov and V. G. Krivokhizhin, Phys. Lett. B **333**, 190 (1994)
- [2] A. Accardi *et al.*, Eur. Phys. J. C **76**, no. 8, 471 (2016)
- [3] A. V. Kotikov, V. G. Krivokhizhin and B. G. Shaikhatdenov, J. Phys. G **42**, no. 9, 095004 (2015)

- [4] A. V. Kotikov, V. G. Krivokhizhin and B. G. Shaikhatdenov, JETP Lett. **101**, no. 3, 141 (2015).
- [5] A. C. Benvenuti *et al.* [BCDMS Collaboration], Phys. Lett. B **223** (1989) 485.
- [6] A. C. Benvenuti *et al.* [BCDMS Collaboration], Phys. Lett. B **237**, 599 (1990).
- [7] A. C. Benvenuti *et al.* [BCDMS Collaboration], Phys. Lett. B **195**, 91 (1987).
- [8] R. S. Thorne, Eur. Phys. J. C **74**, no. 7, 2958 (2014)
- [9] S. Alekhin, J. Blumlein and S. Moch, Phys. Rev. D **89**, no. 5, 054028 (2014)
- [10] P. Jimenez-Delgado and E. Reya, Phys. Rev. D **89**, no. 7, 074049 (2014)
- [11] J. Gao *et al.*, Phys. Rev. D **89**, no. 3, 033009 (2014)
- [12] R. D. Ball *et al.*, Nucl. Phys. B **867**, 244 (2013).
- [13] A. D. Martin, A. J. T. M. Mathijssen, W. J. Stirling, R. S. Thorne, B. J. A. Watt and G. Watt, Eur. Phys. J. C **73**, no. 2, 2318 (2013); L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, Eur. Phys. J. C **75**, no. 5, 204 (2015).
- [14] G. Watt, JHEP **1109**, 069 (2011)
- [15] S. Alekhin, J. Blumlein and S. Moch, Phys. Rev. D **86**, 054009 (2012)
- [16] A. C. Benvenuti *et al.* [BCDMS Collaboration], Phys. Lett. B **195**, 97 (1987).
- [17] A. C. Benvenuti *et al.* [BCDMS Collaboration], Phys. Lett. B **223**, 490 (1989).
- [18] M. Virchaux and A. Milsztajn, Phys. Lett. B **274**, 221 (1992).
- [19] V. Genchev *et al.*, *in* Proc. Int. Conference of Problems of High Energy Physics (1988), Dubna, V.2., p.6.
- [20] V. G. Krivokhizhin and A. V. Kotikov, Phys. Atom. Nucl. **68**, 1873 (2005) [Yad. Fiz. **68**, 1935 (2005)].
- [21] B. G. Shaikhatdenov, A. V. Kotikov, V. G. Krivokhizhin and G. Parente, Phys. Rev. D **81**, 034008 (2010) Erratum: [Phys. Rev. D **81**, 079904 (2010)]
- [22] V. G. Krivokhizhin and A. V. Kotikov, Phys. Part. Nucl. **40**, 1059 (2009).
- [23] A. Abdel-Rehim *et al.*, Phys. Rev. D **92**, no. 11, 114513 (2015) Erratum: [Phys. Rev. D **93**, no. 3, 039904 (2016)]
- [24] K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).
- [25] L. W. Whitlow, E. M. Riordan, S. Dasu, S. Rock and A. Bodek, Phys. Lett. B **282**, 475 (1992).
- [26] L.W. Whitlow, Ph.D. Thesis Stanford University, SLAC report 357 (1990).
- [27] M. Arneodo *et al.* [New Muon Collaboration], Nucl. Phys. B **483**, 3 (1997)
- [28] D. A. Ross and C. T. Sachrajda, Nucl. Phys. B **149**, 497 (1979)l G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B **175**, 27 (1980).
- [29] A. L. Kataev and G. Parente, Phys. Lett. B **566**, 120 (2003)
- [30] A. V. Kotikov and V. N. Velizhanin, hep-ph/0501274.
- [31] A. V. Kotikov, Phys. Part. Nucl. **38**, 1 (2007) Erratum: [Phys. Part. Nucl. **38**, 828 (2007)].
- [32] D. I. Kazakov and A. V. Kotikov, Nucl. Phys. B **307**, 721 (1988) Erratum: [Nucl. Phys. B **345**, 299 (1990)]; A. V. Kotikov, Phys. Atom. Nucl. **57**, 133 (1994) [Yad. Fiz. **57**, 142 (1994)].
- [33] F. J. Yndurain, Phys. Lett. **74B**, 68 (1978).

- [34] A. Gonzalez-Arroyo and C. Lopez, Nucl. Phys. B **166**, 429 (1980); A. Gonzalez-Arroyo, C. Lopez and F. J. Yndurain, Nucl. Phys. B **174**, 474 (1980).
- [35] G. Parisi and N. Surlas, Nucl. Phys. B **151**, 421 (1979); I. S. Barker, C. S. Langensiepen and G. Shaw, Nucl. Phys. B **186**, 61 (1981). I. S. Barker, B. R. Martin and G. Shaw, Z. Phys. C **19**, 147 (1983); I. S. Barker and B. R. Martin, Z. Phys. C **24**, 255 (1984).
- [36] V. G. Krivokhizhin, S. P. Kurlovich, R. Lednický, S. Nemecek, V. V. Sanadze, I. A. Savin, A. V. Sidorov and N. B. Skachkov, Z. Phys. C **48**, 347 (1990); V. G. Krivokhizhin, S. P. Kurlovich, V. V. Sanadze, I. A. Savin, A. V. Sidorov and N. B. Skachkov, Z. Phys. C **36**, 51 (1987).
- [37] A. V. Kotikov, G. Parente and J. Sanchez Guillen, Z. Phys. C **58**, 465 (1993);
- [38] A. L. Kataev, A. V. Kotikov, G. Parente and A. V. Sidorov, Phys. Lett. B **388**, 179 (1996); Phys. Lett. B **417**, 374 (1998); A. L. Kataev, G. Parente and A. V. Sidorov, Nucl. Phys. B **573**, 405 (2000)
- [39] A. L. Kataev, G. Parente and A. V. Sidorov, Phys. Part. Nucl. **34**, 20 (2003) [Fiz. Elem. Chast. Atom. Yadra **34**, 43 (2003)] Erratum: [Phys. Part. Nucl. **38**, no. 6, 827 (2007)] [hep-ph/0106221].
- [40] J. Blumlein, H. Bottcher and A. Guffanti, Nucl. Phys. B **774**, 182 (2007)
- [41] A. V. Kotikov, V. G. Krivokhizhin and B. G. Shaikhatdenov, Phys. Atom. Nucl. **75**, 507 (2012).
- [42] F. James and M. Ross. MINUIT - 1987. CERN Computer Center Library, D 505, Geneve.
- [43] M. Arneodo *et al.* [New Muon Collaboration], Phys. Rev. D **50**, R1 (1994).
- [44] S. Kumano, Phys. Rept. **303**, 183 (1998)
- [45] B. A. Kniehl, A. V. Kotikov, A. I. Onishchenko and O. L. Veretin, Phys. Rev. Lett. **97**, 042001 (2006)
- [46] D. J. Broadhurst, A. L. Kataev and C. J. Maxwell, Phys. Lett. B **590**, 76 (2004)
- [47] S. Alekhin, J. Blumlein, L. Caminadac, K. Lipka, K. Lohwasser, S. Moch, R. Petti and R. Placakyte, Phys. Rev. D **91**, no. 9, 094002 (2015)
- [48] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).
- [49] J. A. M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. B **724**, 3 (2005)
- [50] V. N. Velizhanin, arXiv:1411.1331 [hep-ph]; V. N. Velizhanin, Nucl. Phys. B **860**, 288 (2012)
- [51] P. A. Baikov, K. G. Chetyrkin and J. H. Kuhn, Nucl. Part. Phys. Proc. **261-262**, 3 (2015) doi:10.1016/j.nuclphysbps.2015.03.002 [arXiv:1501.06739 [hep-ph]].
- [52] J. Davies, A. Vogt, B. Ruijl, T. Ueda and J. A. M. Vermaseren, arXiv:1610.07477 [hep-ph].
- [53] G. Grunberg, Phys. Lett. **95B**, 70 (1980) Erratum: [Phys. Lett. **110B**, 501 (1982)]; Phys. Rev. D **29**, 2315 (1984).
- [54] A. V. Kotikov, Phys. Lett. B **338**, 349 (1994); A. V. Kotikov and B. G. Shaikhatdenov, Phys. Atom. Nucl. **78**, no. 4, 525 (2015) [Yad. Fiz. **78**, no. 6, 563 (2015)].
- [55] G. Cvetič, A. Y. Illarionov, B. A. Kniehl and A. V. Kotikov, Phys. Lett. B **679**, 350 (2009); A. V. Kotikov, A. V. Lipatov and N. P. Zotov, J. Exp. Theor. Phys. **101**, 811 (2005) [Zh. Eksp. Teor. Fiz. **128**, 938 (2005)]; A. V. Kotikov and B. G. Shaikhatdenov, Phys. Part. Nucl. **44**, 543 (2013); arXiv:1606.07888 [hep-ph].